$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{\Sigma R_t}$$

where $T_{\infty,1} - T_{\infty,4}$ is the *overall* temperature difference and the summation includes all thermal resistances. Hence

$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{[(1/h_1 A) + (L_A/k_A A) + (L_B/k_B A) + (L_C/k_C A) + (1/h_4 A)]}$$

Alternatively, the heat transfer rate can be related to the temperature difference and resistance associated with each element. For example,

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{(1/h_1 A)} = \frac{T_{s,1} - T_2}{(L_A/k_A A)} = \frac{T_2 - T_3}{(L_B/k_B A)} = \cdots$$

$$\text{Heat flow} = \frac{\text{thermal potential difference}}{\text{thermal resistance}}$$

***Overall heat transfer coefficient**

With composite systems it is often convenient to work with an overall heat transfer coefficient, U, which is defined by an expression analogous to Newton's law of cooling. Accordingly,

$$q_x \equiv UA\,\Delta T$$

Or

$$\boldsymbol{q_x = UA(T_{\infty 1} - T_{\infty 4})}$$

That means:

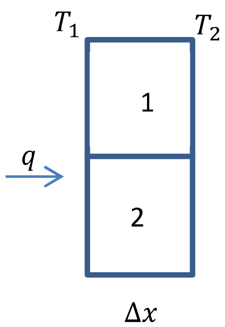
$$UA = 1/R_{tot}.$$

$$U = \frac{1}{R_{tot}A} = \frac{1}{[(1/h_1) + (L_A/k_A) + (L_B/k_B) + (L_C/k_C) + (1/h_4)]}$$

b. Material in parallel:

$$q_x = q_1 + q_2$$

$$q_1 = \frac{T_1 - T_2}{\frac{\Delta x}{k_1 A_1}}$$



$q_2 = \frac{T_1 - T_2}{\frac{\Delta x}{k_2 A_2}}$

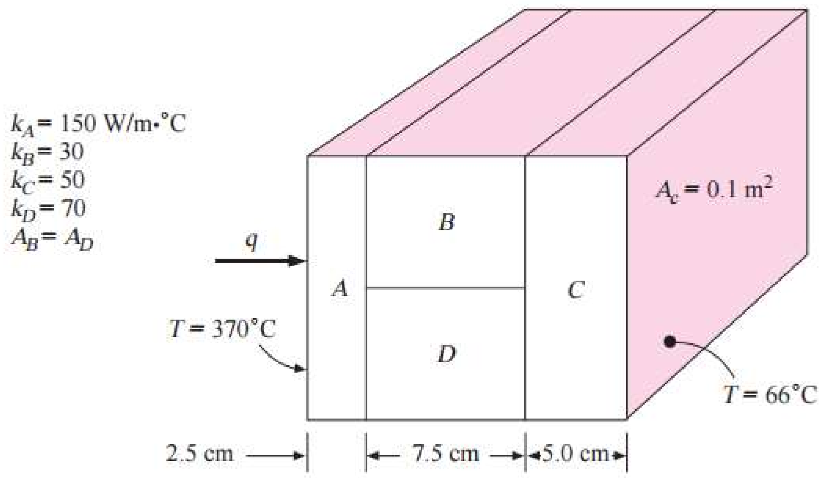
$q_x = (T_1 - T_2)\left[\frac{1}{\frac{\Delta x}{k_1 A_1}} + \right] + \left[\frac{1}{\frac{\Delta x}{k_2 A_2}}\right] = \frac{\Delta T}{R_e}$

$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2}$

Where:

R_e : Equavelant resistance.

Example 2.1: Find the heat transfer per unit area through the composite wall in Figure below. Assume one-dimensional heat flow.



Solution:

$q = \frac{\Delta T}{\sum R}$

$R = \frac{\Delta x}{kA}$

$R_A = \frac{0.025}{(150)(0.1)} = 1.667 * 10^{-3}$

$R_B = \frac{0.075}{(30)(0.05)} = 0.05$

$R_C = \frac{0.05}{(50)(0.1)} = 0.01$

$R_D = \frac{0.075}{(70)(0.05)} = 0.02143$

$R = R_A + R_C + \frac{1}{\frac{1}{R_B} + \frac{1}{R_D}} = 2.667 * 10^{-2}$

$$q = \frac{\Delta T}{R} = \frac{370-66}{2.667*10^{-2}} = 11400 \text{ W}$$

H.W CH2:1,2,3,4

H.W PLANE WALL CH2

2-1 A wall 2 cm thick is to be constructed from material that has an average thermal conductivity of 1.3 W/m · °C. The wall is to be insulated with material having an average thermal conductivity of 0.35W/m · °C, so that the heat loss per square meter will not exceed 1830W. Assuming that the inner and outer surface temperatures of the insulated wall are 1300 and 30°C, calculate the thickness of insulation required.

2-2 A certain material 2.5 cm thick, with a cross-sectional area of 0.1 m^2, has one side maintained at 35 °C and the other at 95°C. The temperature at the center plane of the material is 62°C, and the heat flow through the material is 1 kW. Obtain an expression for the thermal conductivity of the material as a function of temperature.

2-3 A composite wall is formed of a 2.5-cm copper plate, a 3.2-mm layer of asbestos, and a 5-cm layer of fiberglass. The wall is subjected to an overall temperature difference of 560°C. Calculate the heat flow per unit area through the composite structure.

2-5 One side of a copper block 5 cm thick is maintained at 250°C. The other side is covered with a layer of fiberglass 2.5 cm thick. The outside of the fiberglass is maintained at 35°C, and the total heat flow through the copper-fiberglass combination is 52 kW. What is the area of the slab?

2-6 An outside wall for a building consists of a 10-cm layer of common brick and a 2.5-cm layer of fiberglass [k =0.05 W/m · °C]. Calculate the heat flow through the wall for a 25°C temperature differential.

2-7 One side of a copper block 4 cm thick is maintained at 175°C. The other side is covered with a layer of fiberglass 1.5 cm thick. The outside of the fiberglass is maintained at 80°C, and the total heat flow through the composite slab is 300 W. What is the area of the slab?

2-8 A plane wall is constructed of a material having a thermal conductivity that varies as the square of the temperature according to the relation $k = k_0(1+\beta T^2)$. Derive an expression for the heat transfer in such a wall.

2.2.2 Radial system

a. Cylindrical

Consider a long cylinder of inside radius r_i, outside radius ro, and length L, such as the one shown in Figure 2-3. We expose this cylinder to a temperature differential $T_i - T_o$ and ask what the heat flow will be. For a cylinder with length very large compared to diameter, it may be assumed that the heat flows only in a radial direction, so that the only space coordinate needed to specify the system is r.

The general conduction equation in cylindrical coordinate:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \quad (2.6)$$

Assumptions:

- ❖ steady state.
- ❖ One dimension with radius only.
- ❖ No heat generation.

Equation 2.6 will reduced to:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) = 0 \qquad 2.7$$

We may determine the temperature distribution in the cylinder by solving Equation 2.7 and applying appropriate boundary conditions. Assuming the value of k to be constant, Equation 2.7 may be integrated twice to obtain the general solution

$$r\frac{\partial T}{\partial r} = C_1$$

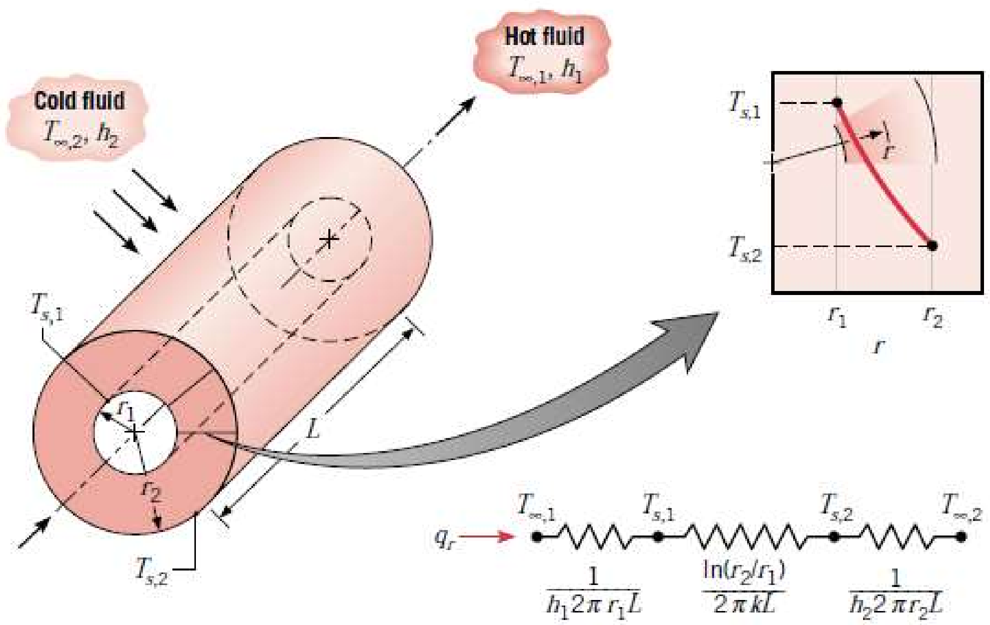


FIGURE 2-3: Hollow cylinder with convective surface conditions.

$$T(r) = C_1 \ln r + C_2$$

To obtain the constants of integration C_1 and C_2, we introduce the following boundary conditions:

$$T(r_1) = T_{s,1} \qquad \text{and} \qquad T(r_2) = T_{s,2}$$

Applying these conditions to the general solution, we then obtain

$T_{s,1} = C_1 \ln r_1 + C_2$ and $T_{s,2} = C_1 \ln r_2 + C_2$

Solving for C_1 and C_2

$$C_1 = \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)}$$

$$C_2 = T_{s,2} - \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)} * \ln(r_2)$$

and substituting into the general solution, we then obtain

$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2} \qquad 2.8$$

The rate at which energy is conducted across any cylindrical surface in the solid may be expressed as

$$q_r = -kA\frac{\partial T}{\partial r} = -k(2\pi r L)\frac{\partial T}{\partial r} \qquad 2.9$$

$$q_r = \frac{2\pi L k(T_{s,1} - T_{s,2})}{\ln(r_2/r_1)} \qquad 2.10$$

From this result it is evident that, for radial conduction in a cylindrical wall, the thermal resistance is of the form

$$R_{t,\mathrm{cond}} = \frac{\ln(r_2/r_1)}{2\pi L k} \qquad \text{K/W, ˚C/W} \qquad 2.11$$

2.2.3 Composite Cylinder Wall

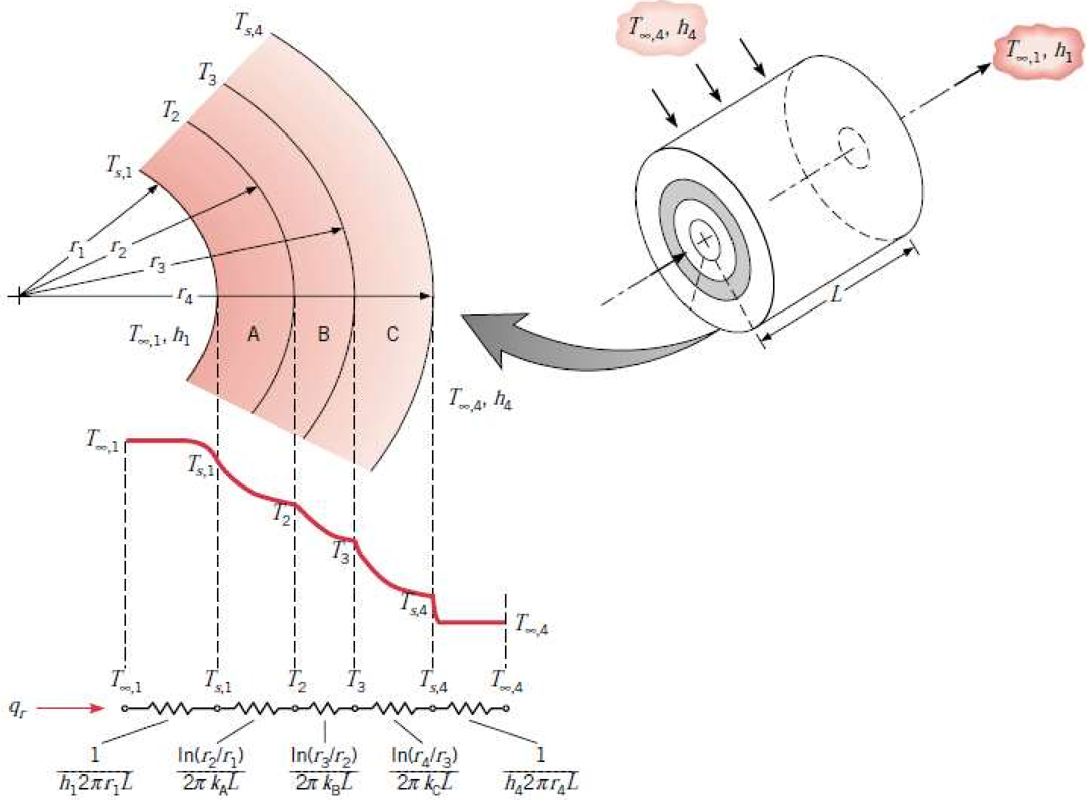


FIGURE 2-4: Temperature distribution for a composite cylindrical wall**.**

Consider now the composite system of Figure 2.4. Recalling how we treated the composite plane wall and neglecting the interfacial contact resistances, the heat transfer rate may be expressed as

$$q_r = \frac{T_{\infty,1} - T_{\infty,4}}{\dfrac{1}{2\pi r_1 L h_1} + \dfrac{\ln(r_2/r_1)}{2\pi k_A L} + \dfrac{\ln(r_3/r_2)}{2\pi k_B L} + \dfrac{\ln(r_4/r_3)}{2\pi k_C L} + \dfrac{1}{2\pi r_4 L h_4}} \qquad 2.12$$

2.2.4 Overall Heat Transfer Coefficient

The foregoing result may also be expressed in terms of an overall heat transfer coefficient. That is,

$$q_r = \frac{T_{\infty,1} - T_{\infty,4}}{R_{tot}} = UA(T_{\infty,1} - T_{\infty,4}) \qquad 2.13$$

If U is defined in terms of the inside area, $A_1 = \boldsymbol{2\pi r_1 L}$, Equations 2.12 and 2.13 may be equated to yield

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{r_1}{k_A}\ln\frac{r_2}{r_1} + \frac{r_1}{k_B}\ln\frac{r_3}{r_2} + \frac{r_1}{k_C}\ln\frac{r_4}{r_3} + \frac{r_1}{r_4}\frac{1}{h_4}} \quad 2.14$$

This definition is arbitrary, and the overall coefficient may also be defined in terms of A_4 or any of the intermediate areas. Note that

$$U_1A_1 = U_2A_2 = U_3A_3 = U_4A_4 = (\Sigma R_t)^{-1} \quad 2.15$$

Example 2.2:

Water flows at 50◦C inside a 2.5-cm-inside-diameter tube such that h_1 = 3500 W/m^2 · °C. The tube has a wall thickness of 0.8 mm with a thermal conductivity of 16 W/m · °C. The outside of the tube loses heat by free convection with h_2 = 7.6 W/m^2 · °C. Calculate the overall heat-transfer coefficient and heat loss per unit length to surrounding air at 20◦C.

Solution

There are three resistances in series for this problem, with L = 1.0 m, d_1= 0.025 m, and d_2 = 0.025 + (2)(0.0008) = 0.0266 m, the resistances may be calculated as:

$$R_1 = \frac{1}{2\pi r_1 L h_1} = \frac{1}{(3500)\pi(0.025)(1)} = 0.00364 \ \text{°C/W.}$$

$$R_{t,cond} = \frac{ln(\frac{r_2}{r_1})}{2\pi k L} = \frac{ln(\frac{0.0266}{0.025})}{2\pi(16)(1)} = 0.00062 \ \text{°C/W.}$$

$$R_2 = \frac{1}{2\pi r_2 L h_2} = \frac{1}{(7.6)\pi(0.0266)(1)} = 1.575 \ \text{°C/W.}$$

$$q_r = U_1A_1(T_{\infty,1} - T_{\infty,2}) = U_2A_2(T_{\infty,1} - T_{\infty,2})$$

$$U_1 = \frac{1}{A_1 R_{total}} = \frac{1}{(\pi d_1 L)R_{total}}$$

$$U_1 = \frac{1}{\pi(0.025)(1)(0.00364+0.00062+1.575)} = 8.064$$

$$U_2 = \frac{1}{A_2 R_{total}} = \frac{1}{(\pi d_2 L)R_{total}}$$

$$U_2 = \frac{1}{\pi(0.0266)(1)(0.00364+0.00062+1.575)} = 7.577$$

$$q_r = 8.064(\pi)(0.025)(1)(50 - 20) = 19 \text{ W}$$

$$q_r = 7.577(\pi)(0.0266)(1)(50 - 20) = 19 \text{ W}$$

Example 2.3:

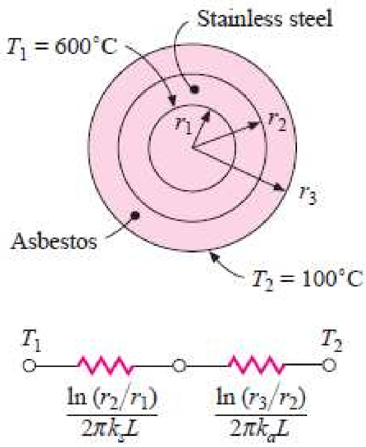
A thick-walled tube of stainless steel [18% Cr, 8% Ni, $k = 19$ W/m · °C] with 2-cm inner diameter (ID) and 4-cm outer diameter (OD) is covered with a 3-cm layer of asbestos insulation [$k = 0.2$ W/m · °C]. If the inside wall temperature of the pipe is maintained at 600°C, calculate the heat loss per meter of length. Also calculate the tube–insulation interface temperature.

$$\frac{q}{L} = \frac{2\pi\,(T_1 - T_2)}{\ln(r_2/r_1)/k_s + \ln(r_3/r_2)/k_a} = \frac{2\pi\,(600-100)}{(\ln 2)/19 + (\ln \frac{5}{2})/0.2} = 680 \text{ W/m}$$

This heat flow may be used to calculate the interface temperature between the outside tube wall and the insulation. We have

$$\frac{q}{L} = \frac{T_a - T_2}{\ln(r_3/r_2)/2\pi k_a} = 680 \text{ W/m}$$

The heat flow is given by



where T_a is the interface temperature, which may be obtained as

$$T_a = 595.8°C$$

The largest thermal resistance clearly results from the insulation, and thus the major portion of the temperature drop is through that material.

H.W conduction in cylinder

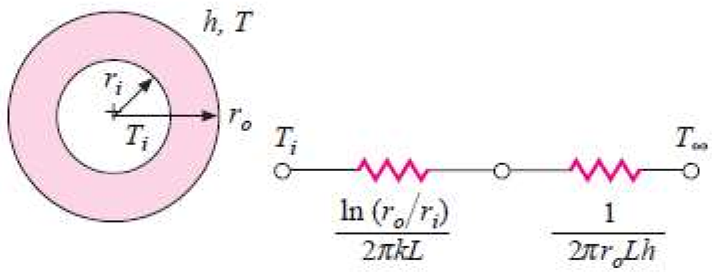
2-31 A5-cm-diameter steel pipe is covered with a 1-cm layer of insulating material having k = 0.22 W/m.˚C followed by a 3-cm-thick layer of another insulating material having k = 0.06 W/m.˚C. The entire assembly is exposed to a convection surrounding condition of h = 60 W/m^2.˚C and T_∞ =15˚C. The outside surface temperature of the steel pipe is 400˚C. Calculate the heat lost by the pipe-insulation assembly for a pipe length of 20 m. Express in Watts.

2-56 Water flows on the inside of a steel pipe with an ID of 2.5 cm. The wall thickness is 2 mm, and the convection coefficient on the inside is 500 W/m^2.°C. The convection coefficient on the outside is 12 W/m^2.°C. Calculate the overall heat-transfer coefficient.

2.2.5 Critical Thickness of Insulation

Let us consider a layer of insulation which might be installed around a circular pipe, as shown in Figure 2-5. The inner temperature of the insulation is fixed at T_i, and the outer surface is exposed to a convection environment at T∞. From the thermal network the heat transfer is

Figure 2-5 Critical insulation thickness.



$$q=\frac{2\pi L\,(T_i-T_\infty)}{\dfrac{\ln(r_o/r_i)}{k}+\dfrac{1}{r_o h}} \qquad 2.16$$

Now let us manipulate this expression to determine the outer radius of insulation ro, which will maximize the heat transfer. The maximization condition is

$$\frac{dq}{dr_o}=0=\frac{-2\pi L\,(T_i-T_\infty)\left(\dfrac{1}{kr_o}-\dfrac{1}{hr_o^2}\right)}{\left[\dfrac{\ln(r_o/r_i)}{k}+\dfrac{1}{r_o h}\right]^2}$$

which gives the result

$$r_o=\frac{k}{h} \qquad 2.17$$

Equation (2.17) expresses the critical-radius-of-insulation concept. If the outer radius is less than the value given by this equation, then the heat transfer will be increased by adding more insulation. For outer radii greater than the critical value an increase in insulation thickness will cause a decrease in heat transfer.

Example 2.4:

Calculate the critical radius of insulation for asbestos [k = 0.17 W/m · ◦C] surrounding a pipe and exposed to room air at 20◦C with h = 3.0 W/m^2 · ◦C. Calculate the heat loss from a 200◦C, 5.0-cm-diameter pipe when covered with the critical radius of insulation and without insulation.

Solution:

From Equation (2-17) we calculate r_o as

$$r_o = \frac{k}{h} = \frac{0.17}{3.0} = 0.0567 \text{ m} = 5.67 \text{ cm}$$

The inside radius of the insulation is 5.0/2 = 2.5 cm, so the heat transfer is calculated from Equation (2.16) as

$$\frac{q}{L} = \frac{2\pi\,(200-20)}{\dfrac{\ln(5.67/2.5)}{0.17} + \dfrac{1}{(0.0567)(3.0)}} = 105.7 \text{ W/m}$$

Without insulation the convection from the outer surface of the pipe is

$$\frac{q}{L} = h(2\pi r)(T_i - T_o) = (3.0)(2\pi)(0.025)(200-20) = 84.8 \text{ W/m}$$

So, the addition of 3.17 cm (5.67 − 2.5) of insulation actually increases the heat transfer by 25 percent. As an alternative, fiberglass having a thermal conductivity of 0.04 W/m · ◦C might be employed as the insulation material. Then, the critical radius would be

$$r_o = \frac{k}{h} = \frac{0.04}{3.0} = 0.0133 \text{ m} = 1.33 \text{ cm}$$

Now, the value of the critical radius is less than the outside radius of the pipe (2.5 cm), so addition of any fiberglass insulation would cause a decrease in the heat transfer.

b. Spherical

Figure 2.6 show a hollow sphere of radius R1 at Ts,1 and R2 at Ts,2. For sphere the heat equation is gives by equation 1.9c